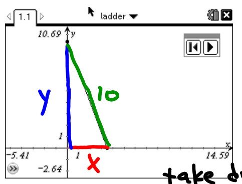
The sliding ladder.

A 10-foot ladder leans against a vertical wall. The base of the ladder is pulled away from the wall at a constant rate of 2 ft/sec. How fast is the top of the ladder falling when t=3?



$\frac{dx}{dt} = 2$ ft/sec

find $\frac{dy}{dt}$ when $t=3$

$x = 2t$

when $t=3, x=6$

$x^2 + y^2 = 10^2$

take der. with respect to "t"

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2\cdot 6\cdot 2 + 2\cdot 8\cdot \frac{dy}{dt} = 0$

$6^2 + y^2 = 10^2$

$y^2 = 64$

$y = 8$

$\frac{dy}{dt} = -\frac{2\cdot 6\cdot 2}{2\cdot 8} = -\frac{12}{8}$

$\frac{dy}{dt} = -\frac{3}{2}$ ft/sec

How fast: $3/2$ ft/sec

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4.6a Related Rates

1. Understand the problem.
2. Develop a mathematical model of the problem.
   Draw a picture, label variables, constants, derivatives
   which der. is given, which der should you find
3. Write an equation relating the variables.
4. Differentiate implicitly. (with respect to time "t")
5. Substitute values for quantities that change with time.
   variables
6. Solve for the unknown rate.

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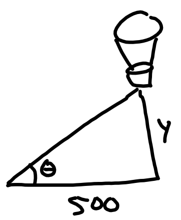
A hot air balloon rising straight up from a level field is tracked by a range finder 500 from the lift-off point. At the moment the range finder's angle of elevation is pi/4, the angle is increasing at the rate of 0.14 radians/~~sec~~ min. How fast is the balloon rising at that moment?



$\frac{d\theta}{dt} = 0.14$ rad/min

find $\frac{dy}{dt}$

$\tan\theta = \frac{y}{500}$

$500\tan\theta = y$

$500\cdot\sec^2\theta\,\frac{d\theta}{dt} = \frac{dy}{dt}$

$500\sec^2\frac{\pi}{4}(0.14) = \frac{dy}{dt}$

$\sec\frac{\pi}{4} = \frac{1}{\cos\frac{\pi}{4}}$

$500\left(\frac{2}{\sqrt{2}}\right)^2(0.14) = \frac{dy}{dt}$

$\sec\frac{\pi}{4} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}$

$500\left(\frac{4}{2}\right)(0.14) = \frac{dy}{dt}$

140 ft/min $= \frac{dy}{dt}$

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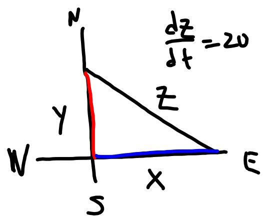
A police cruiser approaching an intersection from the north is chasing a speeding car that has turned the corner and is now moving east. When the cruiser is 0.6 mi north of the intersection and the car is .8 mi east, the distance between the two is increasing at 20 mph. If the cruiser is moving at 60 mph, what is the speed of the car?



$\frac{dz}{dt} = 20$ $\frac{dy}{dt} = -60$ $\frac{dx}{dt} = ?$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(.8)\frac{dx}{dt} + 2\cdot(.6)(-60) = 2\cdot 1\cdot 20$

$.6^2 + .8^2 = z^2$ $z = 1$

$\frac{dx}{dt} = \frac{2\cdot 20 + 2(.6)60}{2(.8)} = 70$

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